Signal-Dependent Film Grain Noise Removal and Generation Based on Higher-Order Statistics

Jacky Chun Kit Yan and Dimitrios Hatzinakos
University of Toronto, Department of Electrical and Computer Engineering
10 King's College Road, Toronto, Ontario, M5S-3G4, Canada
dimitris@comm.toronto.edu

Abstract

In this paper, we propose a new noise filtering scheme that is based on higher-order statistics (H.O.S.) for photographic images corrupted by signal-dependent film grain noise. In addition, reliable estimation of the noise parameter using H.O.S. is proposed. This parameter estimation technique can be used to generate film grain noise which has applications in motion picture and television productions. Simulation results show that the proposed filter perform better than existing methods which are based on second-order statistics.

1. Introduction

Noise suppression is a common application in image processing. Traditionally the noise process is taken to be additive white Gaussian and statistically independent of the signal. However, in the case of photographic images, it is well known that they contain film grain noise that is signal-dependent, that is, the noise statistics depend on the signal. The noise model describing film grain noise and zero mean Gaussian measurement noise w has the form [1]:

$$r(x,y) = s(x,y) + ks^p(x,y)n(x,y) + w(x,y) \qquad (1)$$

where s is the noiseless image measured in density, k is the scanning constant, p is an exponent that depends on film, n is a Gaussian noise with zero mean and unit variance. Because conventional filtering techniques assume a different noise model, they do not perform well in this case.

Existing methods for suppressing signal-dependent film grain noise include modifications of standard techniques designed for additive noise. Examples are the Wiener filter [1] and statistical estimators [2] for the above noise model. These techniques yield better performance than those assuming a false signal-independent noise. However, the above methods assume the parameter k is known *a prior*. Moreover, due to the nonlinearity in the noise model, expressions for the statistical estimators (MMSE and MAP) have a complicated form even in the special case of Gaussian image s, and involve solving a polynomial equation and numerical integration at every pixel.

In this paper, reliable estimation of noise model parameter using higher-order statistics (H.O.S.) is considered. In this respect a new filter (Wiener type) based on H.O.S. is proposed. Also, realistic film grain noise generation, which has applications in television and motion picture productions, becomes possible. Because measurement noise is Gaussian, higher-order statistics of the observed image r would contain contributions from the non-Gaussian image s and signal-dependent noise only, which leads to better parameter estimation. Furthermore, since photographic images are highly non-Gaussian and film grain noise is nonlinearly related to the original image, a lot more information can be extracted from their higher-order statistics [3]. Filtering schemes based on H.O.S. can give better performance.

2. Design of Higher-Order Statistics Based Filter

Assuming the proposed filter $h(x,y)$ be a finite impulse response (FIR) filter with a support region of

$$h(x,y) \neq 0 \qquad for \qquad a \leq x \leq b, c \leq y \leq d, \qquad (2)$$

the filter coefficients $h(x,y)$ can be solved by minimizing a higher-order statistics criterion that is an extension of the mean square error (MSE) criterion used in the correlation based Wiener filter. Let the error signal

$e(x, y)$ be defined as:

$$e(x,y) = s(x,y) - \sum_{i=a}^{b}\sum_{j=c}^{d} h(i,j)r(x-i,y-j). \quad (3)$$

the proposed filter $h(x,y)$ is designed by minimizing the following criterion [4]:

$$J_C^M(h) =$$

$$\left\{ \sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty} Cum^M(e_{xy}, e_{xy}, r_{x-\alpha,y-\beta}, \ldots, \right.$$

$$\left. r_{x-\alpha,y-\beta}) \right\}^2 \geq J_C^M(\hat{h}) \quad (4)$$

with $e_{xy} = e(x,y)$ and $\hat{h}(x,y)$ being the optimum filter based on the cumulant based criterion J_C^M.

3. Cumulant-Based Wiener-Hopf Equation

To compute the optimum filter coefficients, we can extend the correlation based Wiener-Hopf equation and the orthogonality principle to higher-order statistics. By using the idea described in [4], let $\tilde{h}(x,y)$ be the filter satisfying the following cumulant based orthogonality condition:

$$\sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty} Cum^M(e_{xy}, r_{x-i,y-j},$$

$$r_{x-\alpha,y-\beta}, \ldots, r_{x-\alpha,y-\beta}) = 0 \quad (5)$$

Then it can be shown that $\tilde{h}$ is the optimum filter associated with the criterion J_C^M. To derive the cumulant based Wiener-Hopf equation, we start with the orthogonality condition (Eq. (5)), and substitute expression for $e(x,y)$ into Eq. (5) to give:

$$\Rightarrow C_{sr}(p,q) = \sum_{i=a}^{b}\sum_{j=c}^{d} h(i,j)C_{rr}(p-i,q-j) \quad (6)$$

where C_{sr} and C_{rr} are defined as

$$C_{sr}(i,j) = \sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty} Cum^M(s_{xy}, r_{x-i,y-j},$$

$$r_{x-\alpha,y-\beta}, \ldots, r_{x-\alpha,y-\beta}) \quad (7)$$

$$C_{rr}(i,j) = \sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty} Cum^M(r_{xy}, r_{x-i,y-j},$$

$$r_{x-\alpha,y-\beta}, \ldots, r_{x-\alpha,y-\beta}) \quad (8)$$

By substituting estimates of C_{rr} and C_{sr} and forming a linear system of equations, filter coefficients $h(i,j)$ can be solved. Note that the above derivation can be applied similarly to moments, i.e., moment based criterion leading to moment based orthogonality condition and moment based Wiener-Hopf equation.

4. Estimation of Higher-Order Statistics

The use of Eq. (6) requires that C_{sr} and C_{rr} are known. In practice, higher-order cumulants are estimated by replacing the expectation operator by sample averaging over the data. Estimating C_{rr} is not difficult since we have access to the observed signal. However, C_{sr} is not easy to obtain unless we know the signal exactly, which is impossible. Thus we need to determine their relationships with the original signal statistics C_{ss}, which is assumed known.

By substituting the noise model (Eq. (1)) into the definitions of C_{rr} and C_{sr}, we have for $M = 3$ and $p = 0.5$:

$$C_{rr}(i,j) = C_{sr}(i,j) + k^2(R_s(i,j) - m_s^2)$$

$$+ k^2 \sum_{\alpha}\sum_{\beta}(R_s(\alpha,\beta) - m_s^2)\delta(i,j) \quad (9)$$

and

$$C_{sr}(i,j) = \sum_{\alpha}\sum_{\beta} Cum(s_{xy}, s_{x-i,y-j}, s_{x-\alpha,y-\beta})$$

$$+ k^2(R_s(i,j) - m_s^2)$$

$$+ E[s(x,y) - m_s]\sum_{\alpha}\sum_{\beta} R_w(i-\alpha, j-\beta) \quad (10)$$

where R_s and m_s are the autocorrelation and the mean of the signal s respectively:

$$R_s(i,j) = E[s(x,y)s(x-i,y-j)] \quad (11)$$

5. Parameter Estimation

The calculations of Eqs. (9) and (10) require that the constant k be known. However, when this information is not available, we must estimate the constant from the observed image statistics and the *a prior* ideal image statistics. In the case of $p = 0.5$, the variance, skewness, and kurtosis of the received image are related to that of the original image by the following equations:

$$\sigma_r^2 = \sigma_s^2 + k^2 E[s] + \sigma_w^2 \quad (12)$$

$$c_3^r = c_3^s + 3k^2\sigma_s^2 \qquad (13)$$

$$c_4^r = c_4^s + 6k^2c_3^r - 15k^4\sigma_s^2 \qquad (14)$$

The value of k can then be solved by substituting the statistics of the observed image (which can be estimated) and the *a prior* image statistics (σ_s^2, c_3^s, and c_4^s) into any of the above equations. Note from the above equations that the use of higher-order statistics in the presence of Gaussian measurement noise leads to better estimation of k, as cumulants of Gaussian noise are identically zero.

6. Film Grain Noise Generation

As outlined in Introduction, film grain noise generation has applications in television and motion picture productions since digitized film images, video images and computer generated images are routinely combined into one frame. In this process artificial noise is added to the video and computer generated images to match the grain pattern of the film. To generate the right amount of artificial film grain noise, the noise parameter k must be known.

Using the above method to estimate the noise parameter k would require statistics of the corrupted and ideal images. When k is solved, generate and add a noise image to the computer generated image according to Eq. (1), but without the measurement noise w. In cases where the *a prior* ideal image statistics are not known, an approximate solution was proposed in [5] which uses statistics of a filtered image as the ideal image statistics. The observed image r is processed using a sub-optimal filtering technique that requires no knowledge of the noise statistics.

7. Simulation Results

In this section, the above proposed methods in estimation of the parameter k, and signal-dependent film grain noise removal based on H.O.S. are applied. Two test images of size 256 x 256 were used: Lenna and Mountain. Test image 'Mountain' is shown in Fig. 1.

To test the validity of parameter estimation of k, a number of simulations were performed for the following two cases:

- signal-dependent noise only

- mixture of signal-dependent / signal-independent noise

Signal-dependent film grain noise and Gaussian measurement noise are added to the image 'Lenna'. Sample

Figure 1. Test image: 'Mountain'.
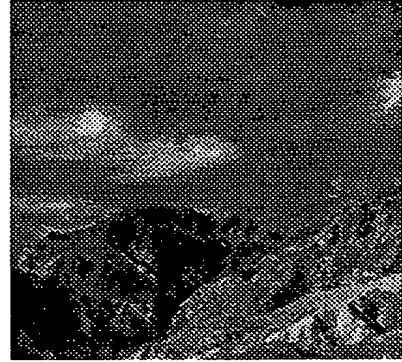
cumulants are calculated using the following relationships:

$$c_1^r = m_1^r \qquad (15)$$

$$c_2^r = \sigma_r^2 = m_2^r - (m_1^r)^2 \qquad (16)$$

$$c_3^r = m_3^r - 3m_1^r m_2^r + 2(m_1^r)^2 \qquad (17)$$

$$c_4^r = m_4^r - 4m_1^r m_3^r - 3(m_2^r)^2 + 12(m_1^r)^2 m_2^r - 6(m_1^r)^4 \qquad (18)$$

The quantities m_1^r, m_2^r, m_3^r, and m_4^r are estimated from an M x N image using sample averaging. The signal statistics σ_s^2, c_3^s, and c_4^s of the image 'Lenna' are known *a prior* and are used to solve for k. The parameter p was fixed to be 0.5 throughout the experiments since this is typical for a variety of film stocks. Because the variance of the signal-independent noise is assumed unknown, k is solved using Eq. (12) with zero measurement noise variance. A value of $k = 0.1$ was selected for moderate noise corruption. The value of k determines the degree of degradation, as can be seen from the variance of the signal-dependent noise term:

$$\sigma_{noise}^2 = k^2 E[s] \qquad (19)$$

Fifty independent runs were performed, with the results summarized below.

Table 1. Estimation of k with true value k=0.1.

σ_w^2	estimated k (mean± standard deviation):		
	2nd order	3rd order	4th order
0	0.0999±0.0015	0.0999±0.0024	0.0998±0.0033
0.05	0.1299±0.0014	0.1003±0.0030	0.1006±0.0041
0.10	0.1939±0.0013	0.1013±0.0039	0.1009±0.0072
0.15	0.2681±0.0015	0.1004±0.0058	0.1007±0.0082
0.20	0.3465±0.0014	0.0990±0.0096	0.0984±0.0159

The advantage of using H.O.S. in estimation is evident from the tables. Second-order statistics results in

high bias and low variance, whereas estimation using H.O.S. has low bias and higher variance. It is clear that estimating k using H.O.S. is better than using second-order statistics in the presence of Gaussian measurement noise.

For noise filtering, again two different cases were investigated:

- signal-dependent noise only

- mixture of signal-dependent/signal-independent noise

A value of $k = 0.1$ was used. For a mixture of noise, variance of measurement noise σ_w^2 was chosen to be 0.005.

The criteria used in evaluating the performance were 1) signal-to-noise ratio (SNR), 2) mean absolute error (MAE), and 3) mean square error (MSE) which is similar to SNR. These are defined below for an image of size M x N:

$$\text{SNR} = 10\log_{10} \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} s^2(x,y)}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\hat{s}(x,y) - s(x,y)]^2} \quad (20)$$

$$\text{MAE} = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\hat{s}(x,y) - s(x,y)] \quad (21)$$

$$\text{MSE} = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\hat{s}(x,y) - s(x,y)]^2 \quad (22)$$

where s and $\hat{s}$ are the ideal and estimated images, respectively. A filter size of 3 x 3 was chosen. Wiener filter designed based on the usual correlation based criterion [1] (designated M_2) and the two proposed higher-order statistics based criteria (third-order moment M_3 and third-order cumulant C_3) are compared.

A number of observations can be made:

- On average higher-order statistics based filters achieved better SNR than the correlation based filter by 1 dB. This improvement may be due to the fact that more information about the image statistics was utilized.

- The cumulant based filter performed as good as the higher-order moment and correlation based filters in the case of signal-dependent noise only. However, for a mixture of noise, the cumulant based filter is not suitable. By examining the cumulant based Wiener-Hopf equation in Eq. (6):

$$C_{sr}(p,q) = \sum_{i=a}^{b}\sum_{j=c}^{d} h(i,j)C_{rr}(p-i,q-j) \quad (23)$$

we have for a mixture of noise:

$$C_{sr}\ (\ mixture\ of\ noise\ )$$
$$\approx C_{sr}\ (\ film\ grain\ noise\ only\ ) \quad (24)$$

and

$$C_{rr}\ (\ mixture\ of\ noise\ )$$
$$\approx C_{rr}\ (\ film\ grain\ noise\ only\ ) \quad (25)$$

thus the cumulant based criterion cannot not recognize the Gaussian measurement noise. The filter performed as if only film grain noise is present.

- Performance of cumulant based filter depends heavily on the properties of the image. For third-order cumulant, if the distribution of an image is close to Gaussian or is symmetric, then it is not appropriate to use the cumulant based filter. Table 5 shows the statistics of the two images. It can be observed that 'Mountain' has the lowest skewness, thus third-order cumulant based filter did not perform well. If fourth-order cumulant is used, the filter should perform well in the case of film grain noise only, as indicated in Table 4.

To test the noise generation procedure, the image 'Lenna' was used for noise generation. Corrupted image was filtered using the method described in [5]. Then k was computed using fourth-order statistics of the two images. Although k can be solved by matching their variances, it was found that variance of the filtered image is lower than that of the corrupted image because edges are blurred to some extent. Thus using variance to obtain k would lead to over-estimation, and the final image would be too noisy. To compare the noise level of the original corrupted and the final image, SNR was used. For the original corrupted image the signal power is the ideal signal power, whereas the signal power of the final image is that of the filtered image. It can be seen from Table 6 that the noise level in two images (noise-added and original corrupted images) are about the same.

8. Conclusions

This paper presents a H.O.S. based filter for filtering images corrupted by signal-dependent film grain noise. In addition, estimation of noise parameter using H.O.S. is proposed and successfully applied in film grain noise generation. Simulation results show that the performance of this filter is better than that of filter based on second-order statistics, and parameter estimation using H.O.S. is more reliable in the presence of Gaussian measurement noise.

Table 2. Test image 'Lenna' with signal-dependent noise only (k=0.1).

	SNR (dB)	MAE	MSE
unfiltered	16.7591	4.6375e-2	3.6820e-3
M_2	19.8458	3.1849e-2	1.8089e-3
M_3	19.9902	3.1212e-2	1.7498e-3
C_3	20.1143	3.0150e-2	1.7005e-3

Table 3. Test image 'Lenna' with mixture of signal-dependent noise (k=0.1) and measurement noise ($\sigma_w^2 = 0.005$).

	SNR (dB)	MAE	MSE
unfiltered	13.0270	7.3938e-2	8.6957e-3
M_2	17.7796	4.1604e-2	2.9110e-3
M_3	17.8464	4.1266e-2	2.8666e-3
C_3	13.7575	6.7819e-2	7.3494e-3

Table 4. Test image 'Mountain' with signal-dependent noise only (k=0.1).

	SNR (dB)	MAE	MSE
unfiltered	15.9772	4.5280e-2	3.4005e-3
M_2	20.9042	2.5644e-2	1.0936e-3
M_3	21.6888	2.1905e-2	9.1283e-4
M_4	21.5968	2.2600e-2	9.3237e-4
C_3	20.7989	2.3356e-2	1.1204e-3
C_4	21.7648	2.2393e-2	8.9398e-4

Table 5. Image statistics of various test images.

	Image Statistics		
Image	mean	variance	skewness
Lenna	3.6363e-1	4.2353e-2	7.2266e-3
Mountain	3.3807e-1	2.0373e-2	1.0025e-3

Table 6. SNR of noise-generated images using different image statistics (original SNR is 16.7620 dB).

statistics	SNR (dB)
second order	16.0929
third order	15.8685
fourth order	16.9721

References

[1] J. F. Walkup and R. C. Choens. Image processing in signal-dependent noise. Opt. Eng., 13:258–266, 1974.

[2] J. F. Walkup G. K. Froehlich and R. B. Asher. Optimal estimation in signal-dependent noise. J. Opt. Soc. Am., 68:1665–1672, 1979.

[3] C. L. Nikias and A. P. Petropulu. Higher-Order Spectral Analysis: A Nonlinear Signal Processing Framework. Prentice Hall, Englewood Cliffs, New Jersey, 1993.

[4] C. C. Feng and C. Y. Chi. Design of wiener filters using a cumulant based mse criterion. Signal Processing, 54:23–48, 1996.

[5] J. C. K. Yan. Statistical methods for film grain noise removal and generation. M. A. Sc. Thesis, University of Toronto, 1997.

Figure 2. Corrupted 'Mountain' with $k = 0.1$.
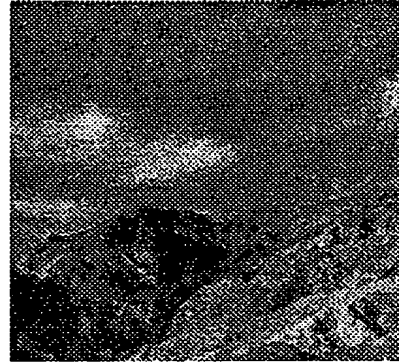

Figure 3. Filtered 'Mountain' using M_2 criterion.
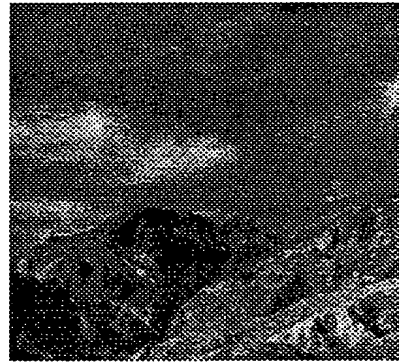
Figure 4. Filtered 'Mountain' using C_4 criterion.